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A Wideband Coaxial Technique for Measuring Permittivity of Materials at Microwave Frequencies

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An analytical solution for the intrinsic impedance of a material is applied to a method of measuring the dielectric and loss tangent with the use of a coaxial test fixture and a vector network analyzer. The use of the equations is best suited for determining the electrical properties of lossy dielectrics over large frequency ranges. The method is unique in that it is not a numerical technique nor does it neglect reflections from any of the material surfaces.

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1. Introduction

An analytical solution for the intrinsic impedance of a material has been found, given the transmitted and reflected transverse electromagnetic (TEM) wave, for the classical three-region problem. This problem is illustrated in figure 1, where regions one and three are air, and region two is the material under test.

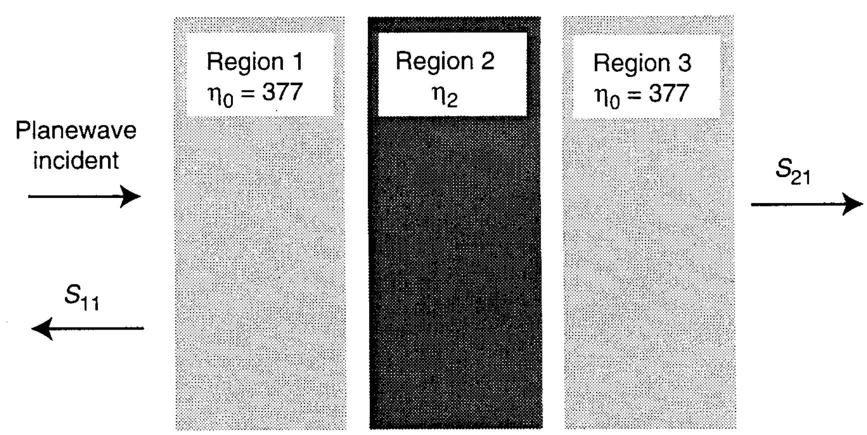
Equations (1) and (2) are the well-known solutions for the transmitted and reflected TEM waves.

$$S_{21} = \frac{4e^{-idk}\eta_0\eta_2}{(\eta_0+\eta_2)^2 \, \dfrac{1-e^{-2idk}(\eta_0-\eta_2)^2}{(\eta_0+\eta_2)^2}} \; . \tag{1}$$

$$S_{11} = \frac{-\eta_0 + \dfrac{\eta_2\left(1+\dfrac{e^{-2idk}(\eta_0-\eta_2)}{\eta_0-\eta_2}\right)}{1-\dfrac{e^{-2idk}(\eta_0-\eta_2)}{\eta_0+\eta_2}}}{\eta_0 + \dfrac{\eta_2\left(1+\dfrac{e^{-2idk}(\eta_0-\eta_2)}{(\eta_0-\eta_2)}\right)}{1-\dfrac{e^{-2idk}(\eta_0-\eta_2)}{\eta_0+\eta_2}}} \; , \tag{2}$$

where d = region length and k = wave number. Mathematica™ was used to solve these equations in terms of the intrinsic impedance of the material η_2; regions one and three have the same intrinsic impedance η_0. This solution is unique [1,2] in that it is not a numerical solution nor does it neglect reflections from the interface of region 2 with region 3. The solution can be used along with coaxial vector network analyzer measurements to provide a simple way of determining the intrinsic impedance and permittivity of nonmagnetic materials. The technique described in this paper is complementary to cavity methods: it is best suited for lossy materials over large

Figure 1. Classical three-region problem: regions one and three are air, and region two is the material under test.



frequency ranges, whereas cavity techniques are better suited for measuring low-loss materials at single frequencies.

2. Calculations

Solving equation (2) for k results in

$$k = -\frac{i}{d}\ln \frac{-\sqrt{-\eta_2^2 + S_{11}\eta_2^2 - 2S_{11}\eta_2\eta_0 + \eta_0^2 + S_{11}\eta_0^2}}{\sqrt{-\eta_2^2 + S_{11}\eta_2^2 + 2S_{11}\eta_2\eta_0 + \eta_0^2 + S_{11}\eta_0^2}} \,. \tag{3}$$

Substituting k from equation (3) into equation (1) gives S_{21} (eq (4)) as a function of S_{11}, η_0, and η_2:

$$S_{21} = \frac{-4\eta_0\eta_2\sqrt{-\eta_2^2 + S_{11}\eta_2^2 + 2S_{11}\eta_2\eta_0 + \eta_0^2 + S_{11}\eta_0^2}}{(\eta_0+\eta_2)^2\sqrt{-\eta_2^2 + S_{11}\eta_2^2 - 2S_{11}\eta_2\eta_0 + \eta_0^2 + S_{11}\eta_0^2}\left(1 - \frac{(\eta_0-\eta_2^2)\left(-\eta_2^2 + S_{11}\eta_2^2 + 2S_{11}\eta_2\eta_0 + \eta_0^2 + S_{11}\eta_0^2\right)}{(\eta_0+\eta_2^2)\left(-\eta_2^2 + S_{11}\eta_2^2 - 2S_{11}\eta_2\eta_0 + \eta_0^2 + S_{11}\eta_0^2\right)}\right)} \,. \tag{4}$$

Now, solving equation (4) for η_2 and letting $\eta_0 = 376.789\ \Omega$ for air results in equation (5), which expresses the intrinsic impedance of the material in terms of the scattering parameters S_{11} and S_{21}:

$$\eta_2 = \frac{376.789\sqrt{-1 - 2S_{11} - S_{11}^2 + S_{21}^2}}{\sqrt{-1 + 2S_{11} - S_{11}^2 + S_{21}^2}} \,. \tag{5}$$

S_{11} and S_{21} in equation (5) are in terms of voltage and are complex numbers. The dielectric constant and loss tangent can then be calculated from the intrinsic impedance, if the material is nonmagnetic. The intrinsic impedance η equals $(z/y)^{1/2}$, where $y = \omega\varepsilon'' + j\omega\varepsilon'$, and $z = \omega\mu'' + j\omega\mu'$ [3]. If we let $\mu'' = 0$ for no magnetic losses and solve for the complex permittivity ε, in terms of permeability (μ') and η, we get

$$\varepsilon' - j\varepsilon'' = \mu'/\eta, \tag{6}$$

where the relative dielectric constant, ε_r, is $\varepsilon'/\varepsilon_0$, and the loss tangent, $\tan\delta$, is $\varepsilon''/\varepsilon'$ [3]. A program was written using PV-wave™ that solves for ε' and ε'' as a function of frequency, using equation (5) to determine the intrinsic impedance and equation (6), with $\mu' = 4\pi \times 10^{-7}$ H/m (free space μ_0), to determine permittivity, given measured S-parameters as a function of frequency.

3. Limitations

When $\tan\delta$ is at or below about 0.1, there are errors at frequencies where the material sample is an integer multiple of a half wavelength because the magnitude of S_{11} is such a small number. But these errors are easily recognized as stray points at integer multiples of a half wavelength. When $\tan\delta$

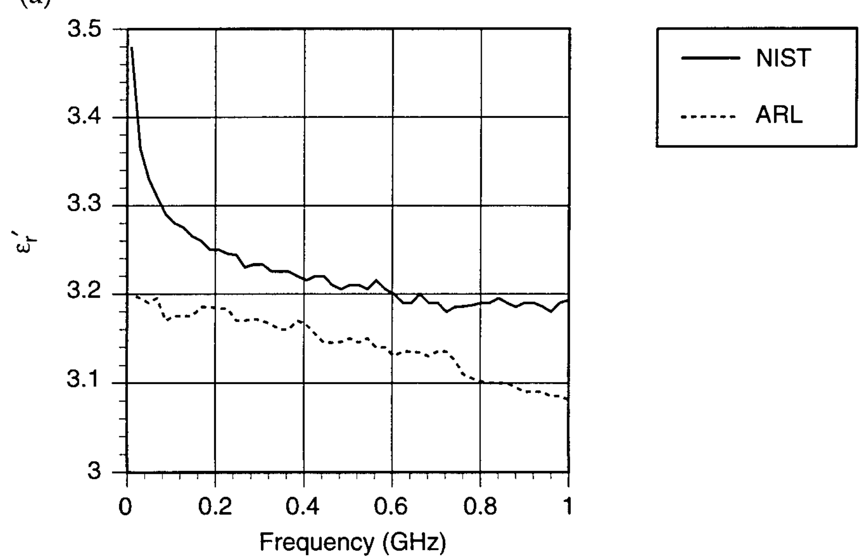
is at or below about 0.01, the imaginary part of permittivity ε_r'' tends to have large error simply because the numbers are so small. Also, the phase of S_{11} cannot be accurately measured for small magnitudes of S_{11}; therefore, if accurate results are desired for the imaginary part of the permittivity, the use of the equations will be limited to lossy dielectric materials ($\tan \delta > 0.1$).

4. Experimental Results

The National Institute of Standards and Technology (NIST) supplied *S*-parameter measurements for two dielectric material samples, cross-linked polystyrene and nylon. A 7-mm-diameter, 10-cm-long air line coaxial fixture was used to make the measurements. The cross-linked polystyrene and nylon were 55 and 15 mm long, respectively. The samples were placed in the air line such that the sample was flush with the port 1 side of the coaxial air line, and *S*-parameter measurements were made on a vector network analyzer from 10 MHz to 1 GHz. The relative permittivities—ε_r' and ε_r'', of cross-linked polystyrene and nylon, respectively—were calculated with the use of the program described above, but with corrections for the phase of S_{21} because the material samples were shorter than the air line. Corrections were also used to account for the small air gaps between the center conductor and the material and the outer conductor and the material [4]. The results compare with results given by NIST, which used its EPSMU software [4], with relatively good agreement, despite the low $\tan \delta$ (<0.1) of both nylon and cross-linked polystyrene (see fig. 2 and 3). The curve in the data near 10 MHz is attributed to the accuracy of the *S*-parameter measurements at these low frequencies. *

Figure 2. Comparison of ARL's calculations and NIST's EPSMU software: (a) ε_r' plotted as a function of frequency for nylon.

(a)



Discussion with M. Janezic of National Institute of Standards and Technology, April 1995.

Figure 2 (cont'd). Comparison of ARL's calculations and NIST's EPSMU software: (b) ε_r'' plotted as a function of frequency for nylon.

(b)

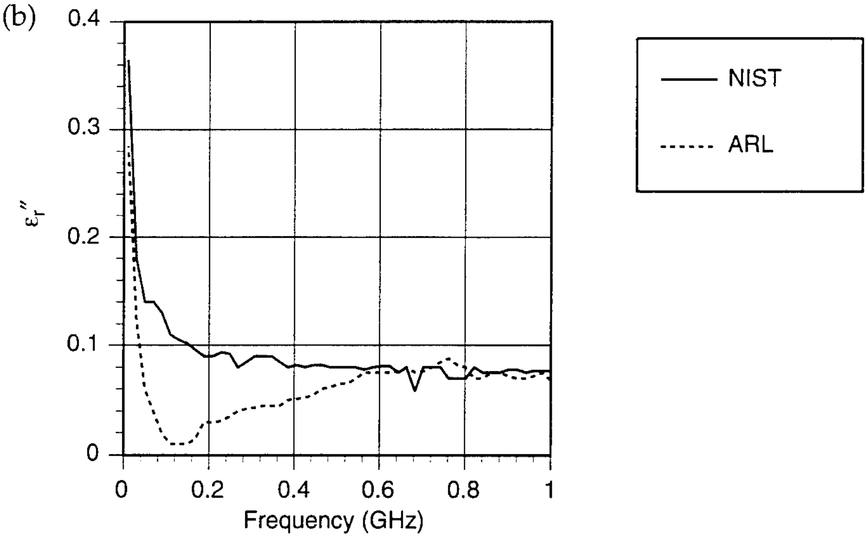
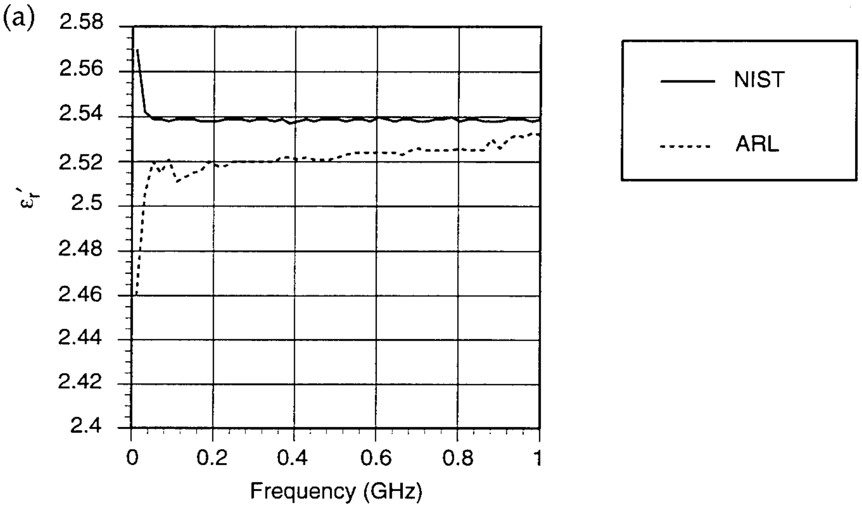
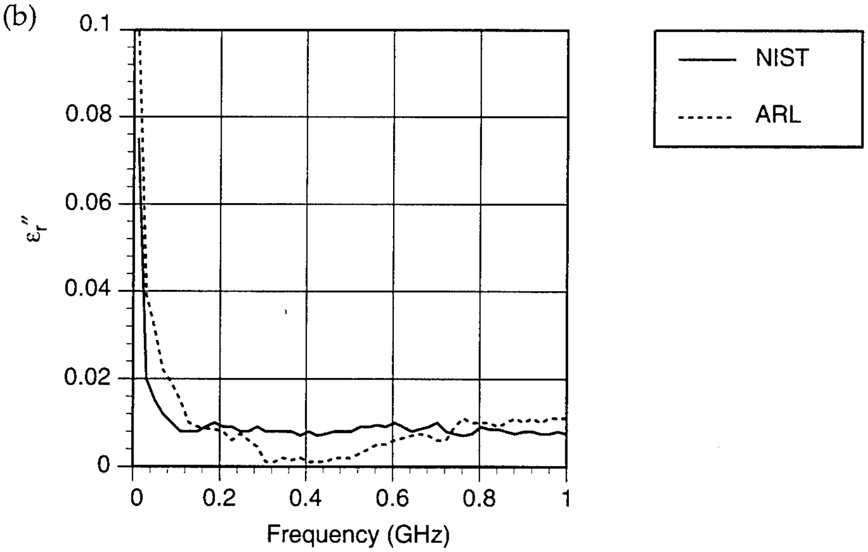


Figure 3. Comparison of ARL's calculations and NIST's EPSMU software: (a) ε_r' and (b) ε_r'' plotted as a function of frequency for crosslinked polystyrene.

(a)



(b)



5. Discussion and Conclusions

Using equation (5), the intrinsic impedance of a material (including magnetic materials) can be calculated from S-parameter measurements of a simple coaxial test fixture containing the material. The complex permittivity can be calculated from the intrinsic impedance using equation (6) if the material is nonmagnetic. The technique is intended to provide a simple way to determine the intrinsic impedance and/or the permittivity of nonmagnetic materials. The equations can be solved on a scientific calculator or, preferably, by a simple program. If accurate results are desired for the loss-tangent, then the measurement technique is limited to lossy dielectrics. One should keep in mind that the measurement is related to the physics of the S-parameter measurements. At frequencies where the material sample is an integer multiple of a half wavelength, there are larger errors, especially if it is a low-loss material. These errors result because, at these frequencies, the magnitude S_{11} tends to be a very small number; also, phase error in the network analyzer measurement increases as the magnitude of the signal decreases. The materials measurement is only as good as the S-parameter measurements.

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